Simultaneous dense coding

2010 J. Phys. A: Math. Theor. 43 055301

(http://iopscience.iop.org/1751-8121/43/5/055301)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **43** (2010) 055301 (10pp) doi:10.1088/1751-8113/43/5/055301

Simultaneous dense coding

Haozhen Situ[1] **and Daowen Qiu**[1,2]

[1] Department of Computer Science, Zhongshan University, Guangzhou 510275, People's Republic of China
[2] SQIG–Instituto de Telecomunicações, IST, TULisbon, Av. Rovisco Pais 1049-001, Lisbon, Portugal

E-mail: issqdw@mail.sysu.edu.cn

Received 28 September 2009, in final form 18 November 2009
Published 12 January 2010
Online at stacks.iop.org/JPhysA/43/055301

Abstract

We present a dense coding scheme between one sender and two receivers, which guarantees that the receivers simultaneously achieve their respective information. In our scheme, the sender first performs a locking operation to entangle the particles from two independent quantum entanglement channels, so that the receivers cannot achieve their information unless they collaborate to perform the unlocking operation. We also show that the quantum Fourier transform can act as the locking operator both in simultaneous dense coding and teleportation. Finally we compare simultaneous dense coding with quantum secret sharing of classical messages.

PACS numbers: 03.67.-a, 03.67.Hk

1. Introduction

Quantum entanglement [1] is the key resource of quantum information theory [2, 3], especially in quantum communication [4]. Sharing an entangled quantum state between a sender and a receiver makes it possible to perform quantum teleportation [5] and quantum dense coding [6]. Quantum teleportation is the process of transmitting an unknown quantum state by using shared entanglement and sending classical information; quantum dense coding is the process of transmitting two bits of classical information by sending part of an entangled state. Teleportation and dense coding are closely related [7, 8] and have been extensively studied in various ways. For example, teleportation and dense coding that use the non-maximally entangled quantum channel have been examined [8–17]; multipartite entangled states have also been considered as the quantum channel [18–26]; another generalization is to perform these two communication tasks under the control of a third party, so-called controlled teleportation and dense coding [27–32].

Recently, a simultaneous quantum state teleportation scheme was proposed by Wang *et al* [33], the aim of which is for all the receivers to simultaneously obtain their respective quantum states from Alice (the sender). In their scheme, Alice first performs a locking operation to entangle the particles from two independent quantum entanglement channels, and therefore the receivers cannot restore their quantum states separately before performing the unlocking operation together. A natural question is whether this idea of locking the entanglement channels adapts for dense coding. The main purpose of this paper is to show that such a locking operator for dense coding really exists. As a result, we propose three simultaneous dense coding protocols which guarantee that the receivers simultaneously achieve their respective information.

The remainder of the paper is organized as follows. In section 2, we introduce three simultaneous dense coding protocols using different entanglement channels. In section 3, we show that the quantum Fourier transform can alternatively be used as the locking operator in simultaneous teleportation. Section 4 contains a comparison between simultaneous dense coding and quantum secret sharing of classical messages. A brief conclusion follows in section 5.

2. Protocols for simultaneous dense coding

Suppose that Alice is the sender, Bob and Charlie are the receivers. Alice intends to send two bits (b_1, b_2) to Bob and another two bits (c_1, c_2) to Charlie under the condition that Bob and Charlie must collaborate to simultaneously find out what she sends.

In the following three subsections, we propose three protocols using the Bell state, GHZ state and W state as the entanglement channels, respectively. The idea of these protocols is to perform the quantum Fourier transform on Alice's qubits before sending them to Bob and Charlie. After receiving Alice's qubits, Bob and Charlie's local states are independent of (b_1, b_2) and (c_1, c_2) so that they know nothing about the encoded bits. Only after performing the inverse quantum Fourier transform together, they can achieve (b_1, b_2) and (c_1, c_2), respectively.

2.1. Protocol 1: using the Bell state

Initially, Alice, Bob and Charlie share two Einstein–Podolsky–Rosen (EPR) pairs [34] $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B}$ and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}$, where qubits A_1A_2 belong to Alice, qubits B and C belong to Bob and Charlie, respectively. The initial quantum state of the composite system is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}. \tag{1}$$

The protocol consists of four steps.

(1) Alice performs unitary transforms $U(b_1b_2)$ on qubits A_1 and $U(c_1c_2)$ on A_2 to encode her bits, like the original dense coding scheme [6]. After that, the state of the composite system becomes

$$|\psi(1)\rangle = U_{A_1}(b_1b_2) \otimes U_{A_2}(c_1c_2)|\psi(0)\rangle = |\phi(b_1b_2)\rangle_{A_1B} \otimes |\phi(c_1c_2)\rangle_{A_2C}, \tag{2}$$

where

$$U(jk) = \sigma_z^k\sigma_x^j, \qquad |\phi(xy)\rangle = \frac{1}{\sqrt{2}}(|0x\rangle + (-1)^y|1\overline{x}\rangle). \tag{3}$$

(2) Alice performs the quantum Fourier transform

$$\mathrm{QFT} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \mathrm{i} & -1 & -\mathrm{i} \\ 1 & -1 & 1 & -1 \\ 1 & -\mathrm{i} & -1 & \mathrm{i} \end{pmatrix} \tag{4}$$

on qubits A_1A_2 to lock the entanglement channels, and then sends A_1 to Bob and A_2 to Charlie. The state of the composite system becomes

$$|\psi(2)\rangle = \mathrm{QFT}_{A_1A_2}[|\phi(b_1b_2)\rangle_{A_1B} \otimes |\phi(c_1c_2)\rangle_{A_2C}]. \tag{5}$$

(3) Bob and Charlie collaborate to perform QFT† on qubits A_1A_2. The state of the composite system becomes

$$|\psi(3)\rangle = \mathrm{QFT}^\dagger_{A_1A_2}|\psi(2)\rangle = |\phi(b_1b_2)\rangle_{A_1B}|\phi(c_1c_2)\rangle_{A_2C}. \tag{6}$$

(4) Bob and Charlie perform the Bell state measurement on qubits A_1B and A_2C, respectively to achieve (b_1, b_2) and (c_1, c_2), like the original dense coding scheme [6].

The following theorem demonstrates that neither Bob nor Charlie alone can distinguish his two-qubit quantum state (i.e. ρ_{A_1B}, ρ_{A_2C}) before step 3. Therefore, they cannot learn the encoded bits from their quantum states unless they collaborate.

Theorem 1. *For each* $b_1, b_2, c_1, c_2 \in \{0, 1\}$, $\rho_{A_1B} = \rho_{A_2C} = I/4$, *where* ρ_{A_1B} *and* ρ_{A_2C} *are the reduced density matrices in subsystems* A_1B *and* A_2C *after step 2 (but before step 3).*

Proof. After step 2, the state of the composite system becomes

$$\begin{aligned}|\psi(2)\rangle &= \mathrm{QFT}_{A_1A_2}\left[\frac{1}{\sqrt{2}}(|0b_1\rangle + (-1)^{b_2}|1\overline{b_1}\rangle)_{A_1B} \otimes \frac{1}{\sqrt{2}}(|0c_1\rangle + (-1)^{c_2}|1\overline{c_1}\rangle)_{A_2C}\right] \\ &= \frac{1}{2}\mathrm{QFT}_{A_1A_2}(|00\rangle \otimes |b_1c_1\rangle + (-1)^{c_2}|01\rangle \otimes |b_1\overline{c_1}\rangle \\ &\quad + (-1)^{b_2}|10\rangle \otimes |\overline{b_1}c_1\rangle + (-1)^{b_2+c_2}|11\rangle \otimes |\overline{b_1c_1}\rangle)_{A_1A_2BC} \\ &= \frac{1}{4}[(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \otimes |b_1c_1\rangle + (-1)^{c_2}(|00\rangle + \mathrm{i}|01\rangle - |10\rangle - \mathrm{i}|11\rangle) \\ &\quad \otimes |b_1\overline{c_1}\rangle + (-1)^{b_2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \otimes |\overline{b_1}c_1\rangle \\ &\quad + (-1)^{b_2+c_2}(|00\rangle - \mathrm{i}|01\rangle - |10\rangle + \mathrm{i}|11\rangle) \otimes |\overline{b_1c_1}\rangle]_{A_1A_2BC}.\end{aligned} \tag{7}$$

The reduced density matrix in subsystem A_1B is

$$\begin{aligned}\rho_{A_1B} &= {}_{A_2C}\langle 0c_1|\psi(2)\rangle\langle\psi(2)|0c_1\rangle_{A_2C} + {}_{A_2C}\langle 0\overline{c_1}|\psi(2)\rangle\langle\psi(2)|0\overline{c_1}\rangle_{A_2C} \\ &\quad + {}_{A_2C}\langle 1c_1|\psi(2)\rangle\langle\psi(2)|1c_1\rangle_{A_2C} + {}_{A_2C}\langle 1\overline{c_1}|\psi(2)\rangle\langle\psi(2)|1\overline{c_1}\rangle_{A_2C} \\ &= \tfrac{1}{4}(|0b_1\rangle\langle 0b_1| + |0\overline{b_1}\rangle\langle 0\overline{b_1}| + |1b_1\rangle\langle 1b_1| + |1\overline{b_1}\rangle\langle 1\overline{b_1}|) \\ &= I/4.\end{aligned} \tag{8}$$

Similarly, the reduced density matrix in subsystem A_2C is also $I/4$. □

2.2. *Protocol 2: using the GHZ state*

Initially, Alice, Bob and Charlie share two Greenberger–Horne–Zeilinger (GHZ) states [35] $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_1B_1B_2}$ and $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_2C_1C_2}$, where qubits A_1A_2 belong to Alice, qubits B_1B_2 and C_1C_2 belong to Bob and Charlie, respectively. The protocol consists of four steps.

(1) Alice performs unitary transforms $U(b_1b_2)$ on qubits A_1 and $U(c_1c_2)$ on A_2 to encode her bits. After that, the state of the composite system becomes

$$|\psi(1)\rangle = |\mathrm{GHZ}(b_1b_2)\rangle_{A_1B_1B_2} \otimes |\mathrm{GHZ}(c_1c_2)\rangle_{A_2C_1C_2}, \tag{9}$$

where

$$|\mathrm{GHZ}(xy)\rangle = \frac{1}{\sqrt{2}}(|0xx\rangle + (-1)^y|1\overline{x}\overline{x}\rangle). \tag{10}$$

(2) Alice performs the quantum Fourier transform on qubits A_1A_2, and then sends A_1 to Bob and A_2 to Charlie.
(3) Bob and Charlie collaborate to perform the inverse quantum Fourier transform on qubits A_1A_2.
(4) Bob and Charlie make the von Neumann measurement using the orthogonal states $\{|\mathrm{GHZ}(xy)\rangle\}_{xy}$ on qubits $A_1B_1B_2$ and $A_2C_1C_2$ respectively to achieve (b_1, b_2) and (c_1, c_2).

The following theorem demonstrates that neither Bob nor Charlie alone can achieve the encoded bits unless they collaborate.

Theorem 2. *$\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are independent of b_1, b_2, c_1, c_2, where $\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are the reduced density matrices in subsystems $A_1B_1B_2$ and $A_2C_1C_2$ after step 2 (but before step 3), respectively.*

Proof. The proof is similar to that of theorem 1. We only point out that $\rho_{A_1B_1B_2} = \rho_{A_2C_1C_2} = \frac{1}{4}(|000\rangle\langle 000| + |011\rangle\langle 011| + |100\rangle\langle 100| + |111\rangle\langle 111|)$. □

2.3. *Protocol 3: using the W state*

Initially, Alice, Bob and Charlie share two W states [25, 36] $\frac{1}{2}(|010\rangle + |001\rangle + \sqrt{2}|100\rangle)_{A_1B_1B_2}$ and $\frac{1}{2}(|010\rangle + |001\rangle + \sqrt{2}|100\rangle)_{A_2C_1C_2}$, where qubits A_1A_2 belong to Alice, qubits B_1B_2 and C_1C_2 belong to Bob and Charlie, respectively. The protocol consists of four steps.

(1) Alice performs unitary transforms $U(b_1b_2)$ on qubits A_1 and $U(c_1c_2)$ on A_2 to encode her bits. After that, the state of the composite system becomes

$$|\psi(1)\rangle = |W(b_1b_2)\rangle_{A_1B_1B_2} \otimes |W(c_1c_2)\rangle_{A_2C_1C_2}, \tag{11}$$

where

$$|W(xy)\rangle = \tfrac{1}{2}(|x10\rangle + |x01\rangle + (-1)^y\sqrt{2}|\overline{x}00\rangle). \tag{12}$$

(2) Alice performs the quantum Fourier transform on qubits A_1A_2, and then sends A_1 to Bob and A_2 to Charlie.
(3) Bob and Charlie collaborate to perform the inverse quantum Fourier transform on qubits A_1A_2.
(4) Bob and Charlie make the von Neumann measurement using the orthogonal states $\{|W(xy)\rangle\}_{xy}$ on qubits $A_1B_1B_2$ and $A_2C_1C_2$ respectively to achieve (b_1, b_2) and (c_1, c_2).

The following theorem demonstrates that neither Bob nor Charlie alone can achieve the encoded bits unless they collaborate.

Theorem 3. *$\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are independent of b_1, b_2, c_1, c_2, where $\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are the reduced density matrices in subsystems $A_1B_1B_2$ and $A_2C_1C_2$ after step 2 (but before step 3), respectively.*

Proof. The proof is similar to that of theorem 1. We only point out that

$$\rho_{A_1B_1B_2} = \rho_{A_2C_1C_2} = \tfrac{1}{8}[2|000\rangle\langle 000| + |001\rangle(\langle 001| + \langle 010|) + |010\rangle(\langle 001| + \langle 010|) + 2|100\rangle\langle 100| + |101\rangle(\langle 101| + \langle 110|) + |110\rangle(\langle 101| + \langle 110|)]. \tag{13}$$

□

2.4. Locking operator

The locking operator used in simultaneous teleportation [33] is

$$U(LOCK)_{12} = H_1 CNOT_{12}, \tag{14}$$

where H is the Hadamard transform, $CNOT$ is the controlled-*NOT* gate, qubit 1 is the control qubit and qubit 2 is the target qubit.

We note that $U(LOCK)$ is not suitable for simultaneous dense coding. To explain the reason, we calculate the reduced density matrix in subsystem A_1B after $U(LOCK)$ is performed when Bell states are used as the entanglement channels. The situations of using GHZ and W states as entanglement channels are similar.

After a calculation similar to that in theorem 1, we have

$$\rho_{A_1B} = \tfrac{1}{4}(|0b_1\rangle\langle 0b_1| + |0b_1\rangle\langle 1b_1| + |0\overline{b_1}\rangle\langle 0\overline{b_1}| - |0\overline{b_1}\rangle\langle 1\overline{b_1}| + |1b_1\rangle\langle 0b_1| + |1b_1\rangle\langle 1b_1| - |1\overline{b_1}\rangle\langle 0\overline{b_1}| + |1\overline{b_1}\rangle\langle 1\overline{b_1}|). \tag{15}$$

Since ρ_{A_1B} is only dependent on b_1, we denote it as $\rho_{A_1B}(b_1)$. We have

$$\rho_{A_1B}(0) = \frac{1}{4}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \rho_{A_1B}(1) = \frac{1}{4}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

After step 2, Bob can distinguish these two states and achieve b_1 by a POVM measurement on qubits A_1B because $\rho_{A_1B}(0)\rho_{A_1B}(1) = 0$. Similarly, Charlie can also achieve c_2 by a POVM measurement on qubits A_2C. Each receiver can achieve 1 bit of his information before they agree to simultaneously find out what Alice sends. The aim of simultaneous dense coding is not achieved when $U(LOCK)$ is used instead of the quantum Fourier transform.

3. Simultaneous teleportation using quantum Fourier transform

In this section, we show that the quantum Fourier transform can alternatively be used as the locking operator in simultaneous teleportation. Let us begin with a brief review of simultaneous teleportation between one sender and two receivers [33]. Suppose that Alice intends to teleport $|\varphi_1\rangle_{T_1} = \alpha_1|0\rangle_{T_1} + \beta_1|1\rangle_{T_1}$ to Bob and $|\varphi_2\rangle_{T_2} = \alpha_2|0\rangle_{T_2} + \beta_2|1\rangle_{T_2}$ to Charlie under the condition that Bob and Charlie must collaborate to simultaneously obtain their respective quantum states. Initially, Alice, Bob and Charlie share two EPR pairs $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B}$ and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}$, where qubits A_1A_2 belong to Alice, qubits B and C belong to Bob and Charlie, respectively. Then the initial quantum state of the composite system is

$$|\chi(0)\rangle = |\varphi_1\rangle_{T_1} \otimes |\varphi_2\rangle_{T_2} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}. \quad (16)$$

The scheme of simultaneous teleportation consists of five steps.

(1) Alice performs the unitary transform $U(LOCK)$ on qubits A_1A_2 to lock the entanglement channels. After that, the state of the composite system becomes

$$\begin{aligned} |\chi(1)\rangle = {} & |\varphi_1\rangle_{T_1} \otimes |\varphi_2\rangle_{T_2} \otimes U(LOCK)_{A_1A_2} \\ & \times \left[\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}\right]. \end{aligned} \quad (17)$$

(2) Alice performs the Bell state measurement on qubits A_1T_1 and A_2T_2, like the original teleportation scheme [5]. It is easy to prove that $|\chi(1)\rangle$ can be written as

$$\begin{aligned} |\chi(1)\rangle = {} & \frac{1}{4}\sum_{x_1=0}^{1}\sum_{y_1=0}^{1}\sum_{x_2=0}^{1}\sum_{y_2=0}^{1} |\phi(x_1y_1)\rangle_{A_1T_1} \otimes |\phi(x_2y_2)\rangle_{A_2T_2} \\ & \otimes U(LOCK)^{\dagger}_{BC}[U_B(x_1y_1)|\varphi_1\rangle_B \otimes U_C(x_2y_2)|\varphi_2\rangle_C]. \end{aligned} \quad (18)$$

If the measurement results are $|\phi(x_1y_1)\rangle_{A_1T_1}$ and $|\phi(x_2y_2)\rangle_{A_2T_2}$, the state of qubits BC collapses into

$$|\chi(2)\rangle = U(LOCK)^{\dagger}_{BC}[U_B(x_1y_1)|\varphi_1\rangle_B \otimes U_C(x_2y_2)|\varphi_2\rangle_C]. \quad (19)$$

(3) Alice sends the measurement results (x_1, y_1) to Bob and (x_2, y_2) to Charlie.

(4) Bob and Charlie collaborate to perform $U(LOCK)$ on qubits BC, and then the state of BC becomes

$$|\chi(3)\rangle = U(LOCK)_{BC}|\chi(2)\rangle = U_B(x_1y_1)|\varphi_1\rangle_B \otimes U_C(x_2y_2)|\varphi_2\rangle_C. \quad (20)$$

(5) Bob and Charlie perform $U(x_1y_1)^{\dagger}$ and $U(x_2y_2)^{\dagger}$ on qubits B and C to obtain $|\varphi_1\rangle$ and $|\varphi_2\rangle$, respectively, like the original teleportation scheme [5].

In the above simultaneous teleportation scheme, $U(LOCK)$ is used to lock the entanglement channels. In section 2.4, we have shown that $U(LOCK)$ is not suitable for simultaneous dense coding, but we find that the quantum Fourier transform can alternatively be used as the locking operator in simultaneous teleportation.

Let us suppose that Alice is the sender, Bob$_i$ $(1 \leqslant i \leqslant N)$ are the receivers. Alice intends to send the unknown quantum states $|\varphi_i\rangle_{T_i} = (\alpha_i|0\rangle + \beta_i|1\rangle)_{T_i}$ to Bob$_i$ under the condition that all the receivers must collaborate to simultaneously obtain $(\alpha_i|0\rangle + \beta_i|1\rangle)_{T_i}$. Initially, Alice and each receiver share an EPR pair $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_iB_i}$. The initial quantum state of the composite system is

$$\begin{aligned} |\chi'(0)\rangle &= \frac{1}{\sqrt{2^N}}\bigotimes_{i=1}^{N}|\varphi_i\rangle_{T_i}\bigotimes_{i=1}^{N}(|00\rangle + |11\rangle)_{A_iB_i} \\ &= \frac{1}{\sqrt{2^N}}\bigotimes_{i=1}^{N}|\varphi_i\rangle_{T_i}\sum_{m=0}^{2^N-1}|m\rangle_{A_1\ldots A_N}|m\rangle_{B_1\ldots B_N}. \end{aligned} \quad (21)$$

The scheme of simultaneous teleportation consists of five steps.

(1) Alice performs the quantum Fourier transform $|j\rangle \rightarrow \frac{1}{\sqrt{2^N}} \sum_{k=0}^{2^N-1} \mathrm{e}^{2\pi \mathrm{ijk}/2^{\mathrm{N}}} |\mathrm{k}\rangle$ on qubits $A_1 \ldots A_N$ to lock the entanglement channels. After that, the state of the composite system becomes

$$
\begin{aligned}
|\chi'(1)\rangle &= \mathrm{QFT}_{\mathrm{A_1 \ldots A_N}} |\chi'(0)\rangle \\
&= \frac{1}{2^N} \bigotimes_{i=1}^{N} |\varphi_i\rangle_{T_i} \sum_{m=0}^{2^N-1} \sum_{k=0}^{2^N-1} \omega^{mk} |k\rangle_{A_1 \ldots A_N} |m\rangle_{B_1 \ldots B_N} \\
&= \frac{1}{2^N} \sum_{k=0}^{2^N-1} \sum_{m=0}^{2^N-1} \omega^{mk} \bigotimes_{i=1}^{N} (|k_i\rangle_{A_i} |\varphi_i\rangle_{T_i}) |m\rangle_{B_1 \ldots B_N},
\end{aligned}
\tag{22}
$$

where k_i is the ith bit of k, $\omega = \mathrm{e}^{2\pi \ \mathrm{i}/2^N}$.

(2) Alice performs the Bell state measurement on each pair of $A_i T_i$.

$$
\begin{aligned}
&\bigotimes_{i=1}^{N} {}_{A_i T_i}\langle \phi(x_i y_i) | \chi'(1)\rangle \\
&= \frac{1}{2^N} \sum_{k=0}^{2^N-1} \bigotimes_{i=1}^{N} {}_{A_i T_i}(\langle 0 x_i| + (-1)^{y_i} \langle 1 \overline{x_i}|)(\alpha_i |k_i 0\rangle + \beta_i |k_i 1\rangle)_{A_i T_i} \frac{1}{\sqrt{2^N}} \sum_{m=0}^{2^N-1} \omega^{mk} |m\rangle_{B_1 \ldots B_N} \\
&= \frac{1}{2^N} \sum_{k=0}^{2^N-1} \prod_{i=1}^{N} [\delta_{k_i 0}(\delta_{x_i 0}\alpha_i + \delta_{x_i 1}\beta_i) + \delta_{k_i 1}(-1)^{y_i}(\delta_{x_i 1}\alpha_i + \delta_{x_i 0}\beta_i)] \mathrm{QFT}_{B_1 \ldots B_N} |k\rangle_{B_1 \ldots B_N} \\
&= \frac{1}{2^N} \mathrm{QFT}_{B_1 \ldots B_N} \bigotimes_{i=1}^{N} [(\delta_{x_i 0}\alpha_i + \delta_{x_i 1}\beta_i)|0\rangle + (-1)^{y_i}(\delta_{x_i 0}\beta_i + \delta_{x_i 1}\alpha_i)|1\rangle]_{B_i} \\
&= \frac{1}{2^N} \mathrm{QFT}_{B_1 \ldots B_N} \bigotimes_{i=1}^{N} U(x_i y_i)(\alpha_i |0\rangle + \beta_i |1\rangle)_{B_i}.
\end{aligned}
\tag{23}
$$

If the measurement result of qubits $A_i T_i$ is $|\phi(x_i y_i)\rangle$, the state of qubits $B_1 \ldots B_N$ collapses into

$$
|\chi'(2)\rangle = \mathrm{QFT}_{B_1 \ldots B_N} \bigotimes_{i=1}^{N} U(x_i y_i) |\varphi_i\rangle_{B_i}.
\tag{24}
$$

(3) Alice sends the measurement result (x_i, y_i) to each Bob$_i$.

(4) All the receivers collaborate to perform QFT† on qubits $B_1 \ldots B_N$, the state of $B_1 \ldots B_N$ becomes

$$
|\chi'(3)\rangle = \mathrm{QFT}^{\dagger}_{B_1 \ldots B_N} |\chi'(2)\rangle = \bigotimes_{i=1}^{N} U(x_i y_i) |\varphi_i\rangle_{B_i}.
\tag{25}
$$

(5) Each Bob$_i$ performs $U(x_i y_i)^\dagger$ on qubit B_i to obtain $|\varphi_i\rangle$.

4. Comparison with quantum secret sharing of classical messages

Quantum secret sharing (QSS), the implementation of the secret sharing problem using quantum information techniques, has been an active area of research in quantum information theory [37–41]. The basic idea of QSS in the simplest case is that Alice wants to distribute a secret (classical message or quantum state) to Bob and Charlie, in such a way that it can be revealed if and only if they collaborate [37]. In a more general case, a secret is distributed among n participants in a way that any k of those participants can reveal the secret, but any set of $k - 1$ or fewer participants contains absolutely no information about the secret. This is called a (k, n) threshold scheme [40]. Not only classical messages but also quantum states can be shared in QSS; hence, two directions have been followed: quantum secret sharing of classical messages (QSSCM) [37–39] and quantum state sharing (QSTS) [40, 41]. In QSSCM, the shared secret is classical information, while in QSTS, the shared secret is an arbitrary unknown quantum state.

Our simultaneous dense coding scheme can be regarded as a (2, 2) threshold QSSCM scheme. Suppose that Alice wants Bob and Charlie to share her N-bit secret. In the secret distributing stage, she first divides the secret into two equal parts, and then sends part 1 to Bob and part 2 to Charlie by running steps 1 and 2 of the simultaneous dense coding protocol $N/4$ times. In the secret revealing stage, Bob and Charlie run steps 3 and 4 of the simultaneous dense coding protocol to achieve part 1 and part 2, respectively. If they put part 1 and part 2 together, the whole secret is revealed. In order to share N bits, $N/2$ EPR pairs are used and $N/2$ qubits are communicated.

From another point of view, if Alice has two different secrets, one for Bob and another for Charlie, she can utilize simultaneous dense coding to guarantee that Bob and Charlie simultaneously reveal their respective secrets. Bob does not know Charlie's secret and vice versa. Obviously this 'simultaneous secret revealing' task is different from secret sharing. For example, Alice wants Bob and Charlie to simultaneously carry out two confidential commercial activities under the condition that the sensitive information of each activity is only revealed to whoever is in charge of that activity.

To sum up, our simultaneous dense coding scheme has two features which are not necessarily acquired in QSSCM schemes:

(1) Each receiver can only reveal his or her part of the secret, which provides a higher level of security.
(2) The receivers can reveal the secret only by the joint unlocking quantum operation, which requires either a quantum channel, shared entanglement, or direct interaction between them. Classical communication does not help the receivers to reveal the secret.

The QSSCM scheme in [37] first established a shared key between Bob and Charlie by measuring a GHZ state and then Alice used this shared key to encode the secret in the secret distributing stage. In the secret revealing stage, Bob and Charlie could obtain the key by classical communication and use it to reveal the secret. In this scheme, there is no way to ensure that each participant can only reveal a designated part of the secret. Thus, the above two features are not acquired in this QSSCM scheme.

5. Conclusion

In summary, we have proposed a simultaneous dense coding scheme between one sender and two receivers, the aim of which is for the receivers to simultaneously achieve their respective information. This scheme may be relevant and useful for improvement of some models or tasks of quantum communication. We have also shown that the quantum Fourier transform, which has been implemented using cavity quantum electrodynamics (QED) [42], nuclear magnetic resonance (NMR) [43–47] and coupled semiconductor double quantum dot (DQD) molecules [48], can act as the locking operator both in simultaneous dense coding and teleportation. Finally we have compared simultaneous dense coding with quantum secret sharing of classical messages.

Acknowledgments

We are very grateful to the referees for their invaluable comments and suggestions that helped to improve the presentation of this paper. This work is supported in part by the National Natural Science Foundation (nos 60573006, 60873055), the Research Foundation for the Doctorial Program of Higher School of Ministry of Education (no. 20050558015), Program for New Century Excellent Talents in University (NCET) of China, and D Qiu is partially funded by project of SQIG at IT, and by FCT and EU FEDER PTDC/EIA/67661/2006.

References

[1] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[2] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[3] Alber G *et al* 2001 *Quantum Information: An Introduction to Basic Theoretical Concepts and Experiments (Springer Tracts in Modern Physics* vol 173*)* (Berlin: Springer)
[4] Gisin N and Thew R 2007 *Nat. Photonics* **1** 165
[5] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[6] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[7] Werner R F 2001 *J. Phys. A: Math. Gen.* **34** 7081
[8] Hao J C, Li C F and Guo G C 2000 *Phys. Lett.* A **278** 113
[9] Li W L, Li C F and Guo G C 2000 *Phys. Rev.* A **61** 034301
[10] Agrawal P and Pati A K 2002 *Phys. Lett.* A **305** 12
[11] Pati A K and Agrawal P 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** S844
[12] Gordon G and Rigolin G 2006 *Phys. Rev.* A **73** 042309
[13] Barenco A and Ekert A K 1995 *J. Mod. Opt.* **42** 1253
[14] Hausladen P, Jozsa R, Schumacher B, Westmoreland M and Wooters W K 1996 *Phys. Rev.* A **54** 1869
[15] Bowen G 2001 *Phys. Rev.* A **63** 022302
[16] Mozes S, Oppenheim J and Reznik B 2005 *Phys. Rev.* A **71** 012311
[17] Pati A K, Parashar P and Agrawal P 2005 *Phys. Rev.* A **72** 012329
[18] Marinatto L and Weber T 2000 *Found. Phys. Lett.* **13** 119
[19] Gorbachev V N and Trubilko A I 2000 *J. Exp. Theor. Phys.* **91** 894
[20] Gorbachev V N, Trubilko A I, Rodichkina A A and Zhiliba A I 2003 *Phys. Lett.* A **314** 267
[21] Joo J, Park Y J, Oh S and Kim J 2003 *New J. Phys.* **5** 136
[22] Bose S, Vedral V and Knight P L 1998 *Phys. Rev.* A **57** 822
[23] Liu X S, Long G L, Tong D M and Li F 2002 *Phys. Rev.* A **65** 022304
[24] Bruß D, Lewenstein M, Sen(De) A, Sen U, D'Ariano G M and Macchiavello C 2006 *Int. J. Quantum Inf.* **4** 415
[25] Agrawal P and Pati A 2006 *Phys. Rev.* A **74** 062320
[26] Li L Z and Qiu D W 2007 *J. Phys. A: Math. Gen.* **40** 10871
[27] Karlsson A and Bourennane M 1998 *Phys. Rev.* A **58** 4394
[28] Deng F G, Li C Y, Li Y S, Zhou H Y and Wang Y 2005 *Phys. Rev.* A **72** 022338
[29] Man Z X, Xia Y J and An N B 2007 *Phys. Rev.* A **75** 052306
[30] Long Y X, Qiu D W and Long D Y 2009 *Int. J. Quantum Inf.* **7** 755
[31] Hao J C, Li C F and Guo G C 2001 *Phys. Rev.* A **63** 054301
[32] Luo C L and Ouyang X F 2009 *Int. J. Quantum Inf.* **7** 365
[33] Wang M Y, Yan F L, Gao T and Li Y C 2008 *Int. J. Quantum Inf.* **6** 201
[34] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[35] Greenberger D M, Horne M A and Zeilinger A 1989 *Bell's Theorem, Quanutm Theory, and Conceptions of the Universe* (Dordrecht: Kluwer)
[36] Dür W, Vidal G and Cirac J I 2000 *Phys. Rev.* A **62** 062314
[37] Hillery M, Bužek V and Berthiaume A 1999 *Phys. Rev.* A **59** 1829
[38] Karlsson A, Koashi M and Imoto N 1999 *Phys. Rev.* A **59** 162

[39] Tittel W, Zbinden H and Gisin N 2001 *Phys. Rev.* A **63** 042301
[40] Cleve R, Gottesman D and Lo H K 1999 *Phys. Rev. Lett.* **83** 648
[41] Lance A M, Symul T, Bowen W P, Sanders B C and Lam P K 2004 *Phys. Rev. Lett.* **92** 177903
[42] Scully M O and Zubairy M S 2002 *Phys. Rev.* A **65** 052324
[43] Vandersypen L M K, Steffen M, Breyta G, Yannoni C S, Cleve R and Chuang I L 2000 *Phys. Rev. Lett.* **85** 5452
[44] Fu L, Luo J, Xiao L and Zeng X 2000 *Appl. Magn. Reson.* **19** 153
[45] Weinstein Y S, Pravia M A, Fortunato E M, Lloyd S and Cory D G 2001 *Phys. Rev. Lett.* **86** 1889
[46] Vandersypen L M K, Steffen M, Breyta G, Yannoni C S, Sherwood M H and Chuang I L 2001 *Nature* **414** 883
[47] Dorai K and Suter D 2005 *Int. J. Quantum Inf.* **3** 413
[48] Dong P, Yang M and Cao Z L 2008 *Phys. Lett.* A **373** 30